LMS and RLS adaptive equalizers in frequency-selective fading channel

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Abstract ---- Linear adaptive equalizers are widely used in wireless communication systems in order to reduce the effects of the channel distortion.

In this paper 16-QAM modulations with LMS and RLS linear adaptive equalizers are compared in the frequency selective Rayleigh fading channel.

Keywords ---- Adaptive equalizers, least mean square (LMS), Recursive least squares (RLS), ISI, Frequency selective fading channel, Quadrature Amplitude Modulation (QAM).

I. INTRODUCTION

In wireless communication channel, the induced Inter-Symbol Interference (ISI) is undesirable distortion, which causes higher error rates. The solution to the ISI problem is to design a receiver that compensates or reduces the ISI in the received signal. An adaptive equalizer is the best compensator for the ISI problem.

To handle with the concept of adaptive equalizers we should focus first on the important terms in the wireless communications, the nature of multipath fading channel and the reasons that cause ISI.

II. MULTIPATH PROPAGATION AND FADING

Radio waves propagate as travelling electromagnetic waves. Because of reflection, diffraction and scattering a radio wave signal arrives at a mobile receiver from different directions with different time delays. This is called multipath propagation of a radio wave signal. [1]

These signal multipath components generally have different amplitudes and carrier-phase offsets and, hence, they may add constructively to strengthen each other's or may add sometimes destructively to attenuate each other's, resulting in a phenomenon called signal fading. Hence, signal fading is a result of multipath signal propagation. [2]

As a mobile receiver moves from one location to another, the phase relationship between the various incoming waves also changes. Thus, there are substantial amplitude and phase fluctuations, and the signal is subjected to slow or fast fading. [1]

A. The slow fading

When we focus on a distance of a couple of kilometers for a mobile receiver, we observe that signal power at this mobile receiver fluctuates around a mean value and the fluctuations have a somewhat longer period. This is referred to as long-term or slow fading. [1]

B. The fast fading

When we concentrate and examine the signal power at this moving receiver over a few hundred meters, we find that signal power fluctuates more rapidly. These rapid fluctuations are caused by a local multipath. The phenomenon giving rise to these rapid fluctuations is referred to as short-term or fast fading. The mobile radio signal contains a short-term fast fading signal superimposed on a long-term slow fading signal. [1]

C. The Doppler shift

Moreover, the speed that the mobile (automobile, train, etc.) is traveling results in frequency offset of the various frequency components of the received signal. This phenomenon is called Doppler shift fd. [2]

III. MULTIPATH FADING CHANNEL AND PDF

In the wireless communication with fading phenomenon the channel is characterized as Multipath Fading Channel. This fading channel can be classified according to its probability density function to a Rayleigh or a Rician fading channel.

D. Rayleigh fading channel

For locations that are heavily shadowed by surrounding buildings and there is no direct path between transmitter and receiver (NLOS Non-Line-Of-Sight path), it is found that a Rayleigh distribution approximates the probability density function (PDF) of this fading channel. [1]

E. Rician fading channel

For locations where there is one path (LOS Line-Of-Sight path) making a dominant contribution to the received signal, such as when the base station is visible to the mobile station, the distribution function is typically found to be that of a Rician distribution. [1]

IV. MULTIPATH FADING CHANNEL AND ISI

When the channel is viewed in the frequency domain, the parameter that may be used to classify the multipath fading channel is the channel’s coherence bandwidth (foherence = fo), which is defined as the frequency band over which the attenuation remains nearly constant and provides a frequency
region where all frequency components behave identically. [1]

**F. The frequency selective channel**

A multipath fading channel is said to be frequency selective if the coherence bandwidth of the channel \( f_0 = 1/T_m \) is small compared with the bandwidth of the transmitted signal \( W = 1/T_s \). In such a situation, the channel has a filtering effect. [3]

\[
\begin{align*}
S(f) & \downarrow \text{Dual functions} \uparrow \\
0 & \rightarrow T_m \quad \text{Maximum excess delay} \\
\text{(a) Multipath intensity profile} \\
\end{align*}
\]

\[
\begin{align*}
S(v) & \downarrow \text{Dual functions} \uparrow \\
f_0 - f_d & \rightarrow f_0 \quad \text{Spectral broadening} \\
\text{(d) Doppler power spectrum} \\
\end{align*}
\]

\[
\begin{align*}
|R(\Delta f)| & \downarrow \text{Dual functions} \uparrow \\
f_0 & \rightarrow 0 \quad \text{Coherence bandwidth} \\
\text{(b) Spaced-frequency correlation function} \\
\end{align*}
\]

\[
\begin{align*}
|R(\Delta t)| & \downarrow \text{Dual functions} \uparrow \\
T_0 & \rightarrow 0 \quad \text{Coherence Time} \\
\text{(c) Spaced-time correlation function} \\
\end{align*}
\]

**Figure 1: Coherence bandwidth and Coherence time [3]**

\[
\begin{align*}
\text{Transmitted signal} & \rightarrow W \\
\text{f_0} & \rightarrow \text{Channel frequency-transfer function} \\
\text{Frequency} & \rightarrow \\
\text{(a) Typical frequency-selective fading case (f_0 < W)} \\
\end{align*}
\]

\[
\begin{align*}
\text{Transmitted signal} & \rightarrow W \\
\text{f_0} & \rightarrow \text{Channel frequency-transfer function} \\
\text{Frequency} & \rightarrow \\
\text{(b) Typical flat fading case (f_0 > W)} \\
\end{align*}
\]

**Figure 2: frequency-selective and flat fading channel [3]**

A channel is said to exhibit a frequency selective fading if the maximum excess delay \( T_m \) is greater than the symbol time \( T_s \). Such multipath dispersion of the signal yields the same kind of ISI (Intersymbol Interference) distortion that is caused by electronic filter. Another name for this category of fading degradation is channel-induced ISI. [4]

**G. The frequency flat channel**

however, the coherence bandwidth of the channel \( f_0 \) is large compared with the transmitted signal bandwidth \( W \), the fading is said to be frequency nonselective, or frequency flat. [3].

**V. THE ADAPTIVE EQUALIZER**

As previously mentioned one of the problems in the frequency selective fading channel is ISI (Inter-symbol Interference). Further multipath propagation and movement of the receiver changes continuously the values of \( f_0 \) and \( f_d \), so that these channel changes its characteristics. These channel is characterized by time-varying frequency response characteristics. To compensate for this channel distortion (ISI) with time-varying channel, we may employ a linear filter with adjustable parameters (Adaptive channel equalizer).

An adaptive filter may be understood as a self-modifying digital filter that adjusts its coefficients in order to minimize an error function. This error function, also referred to as the cost function, is a distance measurement between the reference or desired signal and the output of the adaptive filter. [5]

This equalizer is a tapped-delay-line (TDL) filter. The output \( y(k) \) of this tapped-delay-line (TDL) equalizer (filter) in response to the input sequence \( x(k) \) is defined by the discrete convolution sum

\[
y(k) = w_0x(k) + w_1x(k-1) + \cdots + w_Nx(k-N)
\]

\[
= \sum_{i=0}^{N} w_i x(k-i) = w^T x(k),
\]

where \( w_i \) is the weight at the i-th tap and \( N + 1 \) is the total number of taps. The tap weights constitute the adaptive equalizer coefficients.

The adaptation may be achieved by observing the error between the desired pulse shape and the actual pulse shape at the equalizer output, measured at the sampling instants, and then using this error to estimate the direction in which the tap weights of the equalizer should be changed so as to approach an optimum set of values.

Two important types of adaptive equalizers will be discussed in this paper.

**Figure 3: Basic block diagram of an adaptive filter [5]**
H. LMS equalizer

This adaptive equalizer is based on the mean-square error (MSE) criterion. The MSE is a cost function that requires knowledge of the error function e(k) at all time k. For that purpose, the MSE cannot be determined precisely in practice and is commonly approximated by other cost functions. The simpler form to estimate the MSE function is to work with the instantaneous square error (ISE) given by
\[ \xi(k) = e^2(k), \]
where
\[ e(k) = d(k) - y(k) \]  
To find the minimum error we should calculate the gradient vector
\[ \nabla_w \xi(k) = 2e(k)\nabla_w e(k) \]
\[ = 2e(k)\nabla_w [d(k) - w^T x(k)] \]
\[ = -2e(k)x(k). \]  
(4)
The so-called Wiener solution \( w_o \), that minimizes the MSE cost function, is obtained by equating the gradient vector in Equation (4) to zero
\[ w_o = R^{-1}p \]  
where
\[ R = x(k)x^T(k) \]  
(6)
\[ p = d(k)x^T(k) \]  
(7)
Determining the Wiener solution for the MSE problem requires inversion of matrix R, which is hard to implement in real time. One can then estimate the Wiener solution, in a computationally efficient manner, iteratively adjusting the coefficient vector \( w \) at each time instant \( k \), in such a manner that the resulting sequence \( w(k) \) converges to the desired \( w_o \) solution, possibly in a sufficiently small number of iterations.

The so-called steepest-descent scheme searches for the minimum of a given function following the opposite direction of the associated gradient vector. A factor \( \mu/2 \), where \( \mu \) is the so-called convergence factor, adjusts the step size between consecutive coefficient vector estimates, yielding the following updating procedure: \[ w(k) = w(k-1) - \frac{\mu}{2} \nabla_w \xi(k) = w(k-1) + \mu e(k)x(k). \]  
(8)

I. RLS equalizer

In Recursive-Least-Square equalization (RLS) the weighted least-squares function (WLS) is used. WLS is given by
\[ \xi_D(k) = \sum_{i=0}^{k} \lambda^{k-i} [d(i) - w^T x(i)]^2 \]  
where \( 0<\lambda<1 \) is the so-called forgetting factor. The parameter \( \lambda^{k-i} \) emphasizes the most recent error samples (where \( i \approx k \)) in the composition of the deterministic cost function \( \xi_D(k) \), giving to this function the ability of modeling non-stationary processes.

In order to obtain the equations of the conventional RLS algorithm, the deterministic correlation matrix and cross-correlation vector defined in Equations (6) and (7), respectively, are rewitten as
\[ R(k) = x(k)x^T(k) + \lambda R(k-1), \]
\[ p(k) = x(k)d(k) + \lambda p(k-1). \]  
(10)
and substituted in (equation of Wiener solution)
\[ w(k) = R^{-1}(k)p(k) \]
This leads to
\[ w(k) = w(k-1) + e(k)R^{-1}(k)x(k), \]
(12)
In this updating expression, the computational burden for determining the inverse matrix \( R^{-1}(k) \) can be reduced significantly by employing the matrix inversion lemma \[ [5]. \]
\[ R^{-1}(k) = \frac{1}{\lambda} \left[ R^{-1}(k-1) - \frac{R^{-1}(k-1)x(k)x^T(k)R^{-1}(k-1)}{\lambda + x^T(k)R^{-1}(k-1)x(k)} \right] \]  
(13)
VI. MATLAB SIMULATION AND ANALYSIS

In the simulation LMS and RLS equalizers are compared in Rayleigh frequency selective fading channel with 16-QAM modulation.

![Figure 4: Error estimation in LMS](image)

The linear adaptive LMS equalizer has six weights and a step size of 0.03. The linear adaptive RLS equalizer has six weights with forgetting factor of 0.99. The inverse correlation matrix is initialized with the value of 0.1.

First a random training sequence is generated and then 16-QAM modulation in a Rayleigh frequency-selective fading channel is achieved. After demodulation in the receiver the
training signal is used to set the equalizer tap weights of each equalizer.

The simulation program will plot the magnitude of the error estimate (figures 4 and 6) as well the constellation diagram of the received and equalized signals (figures 5 and 7) of both LMS and RLS adaptive equalizers.

The results of Figures 5 and 7 show the constellation diagram where both equalizers have removed the effects of the fading channel. When we compare the results of figures 4 and 6 we find that the LMS algorithm is more computationally efficient as it took 50% of the time to execute the processing loop. However, the training sequence required by the LMS algorithm is 5 times longer.

In figure 4 the training for error estimation of LMS equalizer requires 1000 symbols but in figure 6 the training for error estimation of RLS equalizer requires only 200 symbols.

VII. CONCLUSION

Inter-symbol interference in Rayleigh frequency selective fading channel causes undesired distortion that can be effectively overcome by adapting RLS algorithm at the receiver. An adaptive equalizer employing RLS equalizer is a better option over LMS equalizer.

The RLS algorithm is also very useful in applications where the environment is slowly varying. The price of all these benefits is a considerable increase in the computational complexity of the algorithms.

The LMS algorithm is very popular and has been widely used due to its extreme simplicity but its convergence speed is slower than RLS algorithm.

REFERENCES